Abstract Interpretation

2008년 1월 30일

숙명여대

창병모

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Motivation

우리는 무엇을 해야 하는가?

❖ 컴퓨터 기술자 무엇을 해야 하는가?

The production of reliable software,
its maintenance and
safe evolution year after year (up to 20 to 30 years).

Hardware vs Software

- Hardware
  - The 25 last years, computer hardware has seen its performances multiplied by 10^4 to 10^6;
- Software
  - The size of programs executed by these computers has grown up in similar proportions;
- Example
  - Windows
    - > 30 000 000 lines
    - > 30 000 known bugs

Software development

- Software development
  - Large scale computer programming is very difficult;
  - Reasoning on large programs is very difficult;
  - Errors are quite frequent.

- Idea
  - Use the computer to find programming errors.
  - How can computers be programmed so as to analyze the behavior of software before and without executing it ?
  - This is essential for safety-critical systems
    (for example: planes, trains, launchers, nuclear plants, …)

Program Analysis

프로그램 분석Static Program Analysis

- 프로그램의 실행 성질을 실행 전에 자동으로 안전하게 어림잡는 기술이다[Yi04-05].
  - 실행 전에: 프로그램을 실행시키지 않고
  - 자동으로: 프로그램이 프로그램을 분석
  - 안전하게: 모든 가능성을 포섭하도록
  - 어림잡는: 실제 이외의 것들이 포함됨
    어림잡지 않으면 불가능

Program Analysis Techniques

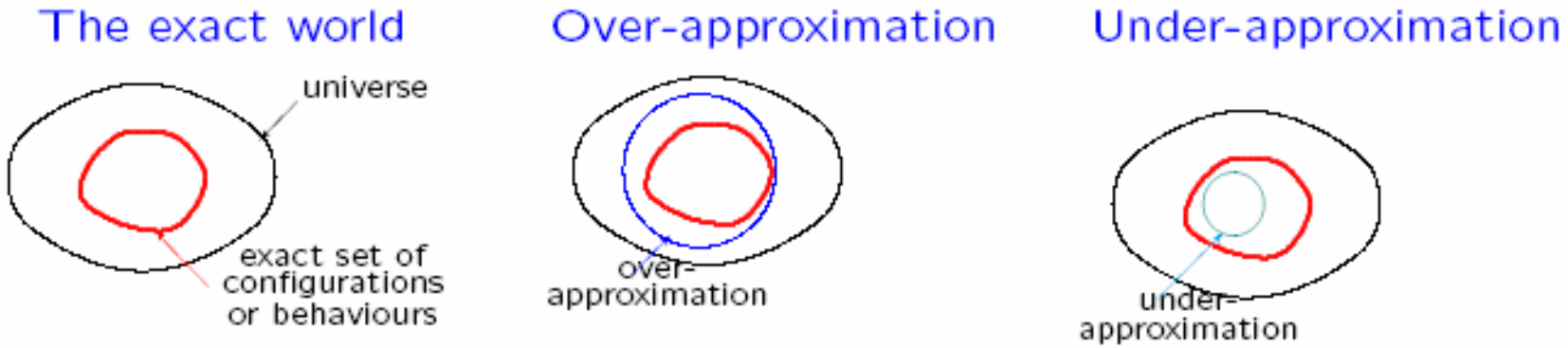
- DFA
  - Conventional dataflow analysis
- Deductive methods
  - The proof size is exponential in the program size!
- Model checking
- Constraint-based analysis
  - Constraint setup and solving
- Type-based analysis
  - Type and effect system
- What else ?

Abstract Interpretation

Abstract interpretation요약 해석

- [Cousot&Cousot77,79]
  - A theory of the approximation of the execution behavior of programs.
- Objective
  - By effective computation of the abstract semantics, it analyzes the behavior of programs before and without executing them.
- 한글로
  - 프로그램 실행 전에 실행하지 않고 그 프로그램의 실행(의미)의 요약본 계산

The Nature of Approximation



- If the approximation is rough enough,
  the abstract semantics is less precise but
  is effectively computable by a computer;

- Because of the information loss,
  not all questions can be definitely answered;

- All answers given by the abstract semantics are always
  correct(sound) with respect to the concrete semantics.

Computable approximation

- 요약(abstract or approximation)이 필요한 이유는 ?
  - 요약 없이는 분석이 끝이 없다
  - 요약 없이 실행해 보면서 모든 경우를 포섭할 수 없다.

- 요약(abstraction) α 예
  - {2,4,6,8, …} → 짝수
  - {-8, -4, -2} → 음수
  - {2, 15, 12, 8} → [2..15]

Example: Sign Analysis

- Source language

  e ::= i | e * e | -e | e + e

- Concrete (execution) semantics

  V : e → Int

  V(i) = i

  V(e1*e2) = V(e1) × V(e2)

  V(-e) = -V(e)

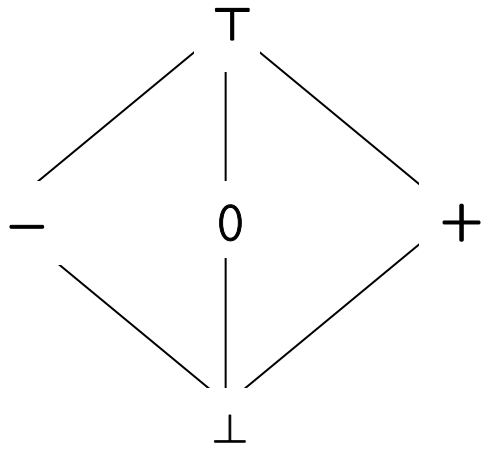
  V(e1+e2) = V(e1) + V(e2)

Sign Analysis

- Example
  - e = -45 * 30 + 23 * -50
  - V(e) = -2500
  - AV(e) = -
- Abstract semantics for sign analysis
  - Define Abstract domain
  - Define Abstract execution

Sign Analysis

- Abstract Domain: Lattice $\hat{A}$



$$2^{\mathbb{Z}} \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \hat{A}$$

- What are $\top$ and $\bot$?

- What are α and γ ?
  - α(abstraction)는 실제 값들을 요약하고
  - γ(concretization) 는 요약된 원소의 실제 의미를 정의한다.

Abstract semantics

- Abstract semantics

  $AV : e \rightarrow \{\bot, -, 0, +, \top\}$

  $$AV(i) = \begin{cases} + & \text{if } i > 0 \\ 0 & \text{if } i = 0 \\ - & \text{if } i < 0 \end{cases}$$

  $AV(-e) = \hat{-} AV(e)$

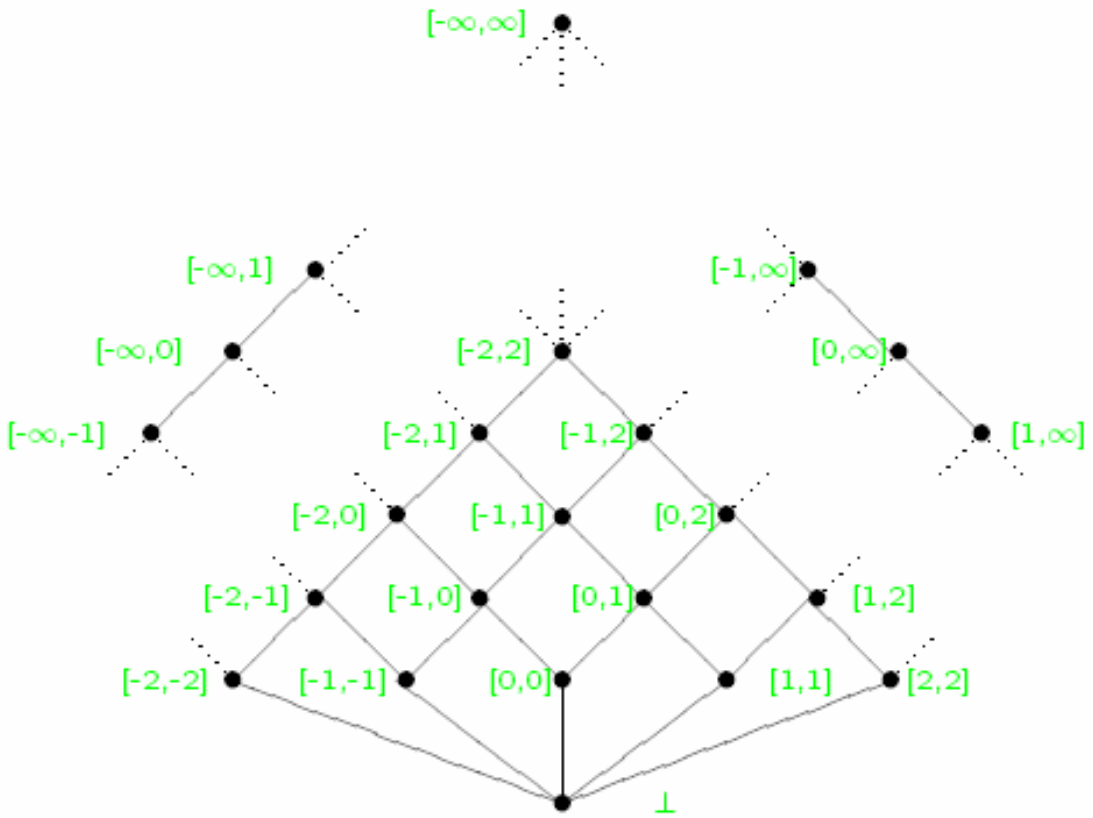
  $AV(e1+e2) = AV(e1) \hat{+} AV(e2)$

  $AV(e1*e2) = AV(e1) \hat{\times} AV(e2)$

- Abstract operator $\hat{a} \hat{+} \hat{b} = ?$
  - $+ \hat{+} + =$
  - $+ \hat{+} - =$
  - $\top \hat{+} - =$

Example: Interval Analysis

The complete lattice **Interval** = (**Interval**, $\sqsubseteq$)



Interval analysis

- Define an abstract semantics that
  - can approximate the concrete (execution) semantics.
- Abstract semantics for interval analysis
  - Interval to approximate concrete values of variables
  - Equation to approximate concrete execution flows.
- Program analysis
  - By computing the least fixpoint of the equation.

Example: interval analysis (1975) [5]

Program to be analyzed:

```
   x := 1;
1:
   while x < 10000 do
2:
        x := x + 1
3:
   od;
4:
```

[5] P. Cousot & R. Cousot, ISOP'76.

Example: interval analysis (1975) [5]

Equations (abstract interpretation of the semantics):

```
   x := 1;
1:
   while x < 10000 do
2:
      x := x + 1
3:
   od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

[5] P. Cousot & R. Cousot, ISOP'1976, POPL'77.

Example: interval analysis (1975) [5]

Increasing chaotic iteration, initialization:

```
    x := 1;
1:
    while x < 10000 do
2:
        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = \emptyset \\ X_2 = \emptyset \\ X_3 = \emptyset \\ X_4 = \emptyset \end{cases}$$

[5] P. Cousot & R. Cousot, ISOP'1976, POPL'77.

Example: interval analysis (1975) [5]

Increasing chaotic iteration:

```
    x := 1;
1:
    while x < 10000 do
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    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration:

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$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,1] \\ X_3 = [2,2] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration:

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    x := 1;
1:
    while x < 10000 do
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        x := x + 1
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    od;
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$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration: **convergence?**

```
    x := 1;
1:
    while x < 10000 do
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        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

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Increasing chaotic iteration: **convergence??**

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Increasing chaotic iteration: **convergence???**

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    x := 1;
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3:
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```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,3] \\ X_3 = [2,4] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration: **convergence????**

```
    x := 1;
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```

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Increasing chaotic iteration: **convergence?????**

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    x := 1;
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    while x < 10000 do
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        x := x + 1
3:
    od;
4:
```

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$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,4] \\ X_3 = [2,5] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration: **convergence??????**

```
    x := 1;
1:
    while x < 10000 do
2:
        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,5] \\ X_3 = [2,5] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Increasing chaotic iteration: **convergence???????**

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$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,5] \\ X_3 = [2,6] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Convergence speed-up by extrapolation:

```
    x := 1;
1:
    while x < 10000 do
2:
        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,+\infty] \quad \Leftarrow \text{widening} \\ X_3 = [2,6] \\ X_4 = \emptyset \end{cases}$$

[5] P. Cousot & R. Cousot, ISOP'1976, POPL'77.

Example: interval analysis (1975) [5]

Decreasing chaotic iteration:

```
    x := 1;
1:
    while x < 10000 do
2:
        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1, +\infty] \\ X_3 = [2, +\infty] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Decreasing chaotic iteration:

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    x := 1;
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Example: interval analysis (1975) [5]

Decreasing chaotic iteration:

```
    x := 1;
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$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,9999] \\ X_3 = [2,+10000] \\ X_4 = \emptyset \end{cases}$$

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Example: interval analysis (1975) [5]

Final solution:

```
    x := 1;
1:
    while x < 10000 do
2:
        x := x + 1
3:
    od;
4:
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,9999] \\ X_3 = [2,+10000] \\ X_4 = [+10000,+10000] \end{cases}$$

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Example: interval analysis (1975) [5]

Result of the interval analysis:

```
    x := 1;
1: {x = 1}
    while x < 10000 do
2: {x ∈ [1, 9999]}
        x := x + 1
3: {x ∈ [2, +10000]}
    od;
4: {x = 10000}
```

$$\begin{cases} X_1 = [1,1] \\ X_2 = (X_1 \cup X_3) \cap [-\infty, 9999] \\ X_3 = X_2 \oplus [1,1] \\ X_4 = (X_1 \cup X_3) \cap [10000, +\infty] \end{cases}$$

$$\begin{cases} X_1 = [1,1] \\ X_2 = [1,9999] \\ X_3 = [2,+10000] \\ X_4 = [+10000,+10000] \end{cases}$$

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Example: interval analysis (1975) [5]

Exploitation of the result of the interval analysis:

```
    x := 1;
1: {x = 1}
    while x < 10000 do
2: {x ∈ [1, 9999]}
        x := x + 1          ⟵ no overflow
3: {x ∈ [2, +10000]}
    od;
4: {x = 10000}
```

[5] P. Cousot & R. Cousot, ISOP'1976, POPL'77.

요약 해석으로 정적 분석

1. 실제 실행 정의(Concrete semantics)
2. 요약 실행 정의(Abstract semantics)
3. 올바른 요약 실행 정의인지 확인
4. 정의된 요약 실행을 계산하는 방법

Concrete Semantics실제 실행

- The concrete (collecting) semantics
  - collects the set of traces(or states) that can reach a given program point
  - may be uncomputable.

- Concrete semantic domain

  CPO (D, ⊑, ⊔)

- Concrete semantic function

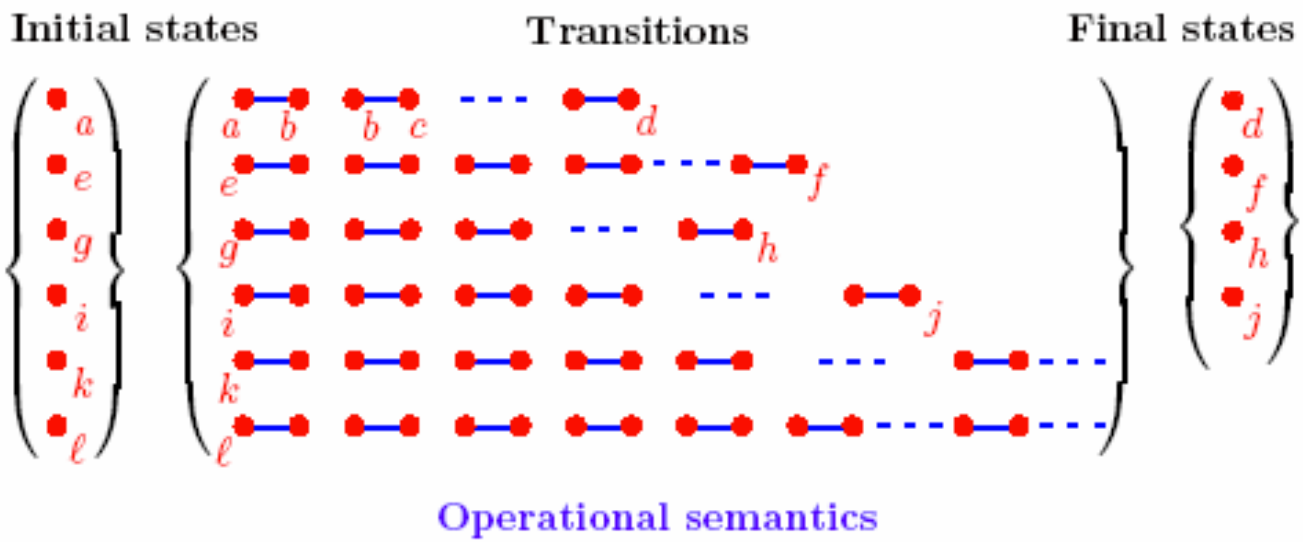
$$F \in D \to D$$

- Concrete semantics

  Least fixpoint of a continuous function $F \in D \to D$

$$lfpF = \bigsqcup_{i \in \mathbb{N}} F^i(\bot_D)$$

Concrete Semantics실제 실행



요약 해석으로 정적 분석

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Abstract Semantics요약 실행

- Abstract semantic domain CPO ($\hat{D}$, ⊑, ⊔)
  - Galois connection $D \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \hat{D}$

- Abstract semantic function
  - Monotone function $\hat{F} \in \hat{D} \to \hat{D}$

$$\forall \hat{x}, \hat{y} \in \hat{D} : x \sqsubseteq y \Rightarrow \hat{F}(x) \sqsubseteq \hat{F}(y)$$

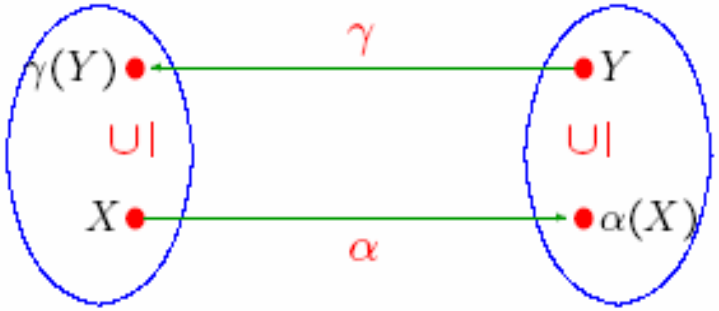
  - Approximation

$$\alpha \circ F \sqsubseteq \hat{F} \circ \alpha, \quad \text{or} \quad F \circ \gamma \sqsubseteq \gamma \circ \hat{F}$$

Galois connection갈로아 연결

- A Galois connection between two sets is a pair of (α, γ) of functions between the sets satisfying

$$X \subseteq \gamma(Y) \quad \Leftrightarrow \quad \alpha(X) \subseteq Y$$



α: abstraction function
γ: concretisation function

Concrete domain D Abstract domain $\hat{D}$

- Abstract values respects the order in the concrete domain

Abstract semantics요약 실행

- Abstract semantics

$$lfp\ \hat{F} = \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\perp})$$

- [Theorem]

  The abstract semantics is an approximation of the concrete semantics.

$$\alpha(lfpF) \sqsubseteq \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\perp}).$$

- Abstract semantics may be computable in finite time or uncomputable.

요약 해석으로 정적 분석

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3. 올바른 요약 실행 정의인지 확인
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Program Analysis

- Program analysis by
  - computing the abstract semantics

    $lfp\ \hat{F} = \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\perp})$ if it is computable in reasonable time

  - or, computing an upper bound $\hat{A}$ of the abstract semantics in finite time

  - then it approximates the concrete execution

$$\alpha(lfpF) \sqsubseteq \hat{A},$$
$$lfpF \sqsubseteq \gamma\hat{A}$$

Widening/Narrowing

- Compute the upperbound by widening and

$$\hat{X}_0 = \hat{\bot}$$

$$\hat{X}_{i+1} = \begin{cases} \hat{X}_i & \text{if } \hat{F}(\hat{X}_i) \sqsubseteq \hat{X}_i \\ \hat{X}_i \bigtriangledown \hat{F}(\hat{X}_i) & \text{otherwise} \end{cases}$$

- Narrowing
  - If the upperbound is $\hat{\mathcal{A}} \overset{\text{let}}{=} \lim_{i \in \mathbb{N}}(\hat{X}_i)$,
  - refine the upperbound $\hat{\mathcal{A}}$ by narrowing

$$\hat{Y}_0 = \hat{\mathcal{A}}$$

$$\hat{Y}_{i+1} = \hat{Y}_i \bigtriangleup \hat{F}(\hat{Y}_i)$$

Applications

응용 사례

- A. Deutsch uses abstract interpretation (including interval analysis) for the static analysis of the embedded ADA software of the Ariane 5 launcher [6];
- Automatic detection of the definiteness, potentiality, impossibility or inaccessibility of run-time errors [7];
- Success for the 502 & 503 flights and the ARD [8].

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6 Flight software (60,000 lines of Ada code) and Inertial Measurement Unit (30,000 lines of Ada code).

7 such as scalar and floating-point overflows, array index errors, divisions by zero and related arithmetic exceptions, uninitialized variables, data races on shared data structures, etc.

8 Atmospheric Reentry Demonstrator: module coming back to earth.

응용 분야

- program transformation & optimization;
- abstract model-checking of infinite systems;
- abstract testing;
- type inference (for undecidable systems);
- mobile code communication topology;

상업화

- First research results: 1975;
- First industrializations:
  -  Connected Components Corporation (U.S.A.), L. Harrison, 1993;
  -  AbsInt Angewandte Informatik GmbH (Germany), R. Wilhelm, 1998;
  -  Polyspace Technologies (France), A. Deutsch & D. Pilaud, 1999.
- Domestic industrializations:
  - Sparrow
    K. Yi, 2007

참고문헌

- Cousot&Cousot
  - [Cousot&Cousot 77, 79]
  - [Cousot&Cousot 92]
  - [Cousot&Cousot 00]
  - …
- 이광근
  - 프로그래밍언어 이야기 I, 2004-2005
  - 관련 논문 리스트
    http://ropas.snu.ac.kr/~kwang/4541.664A/07/papers.html
- Nielson, Nielson and Hankin
  - Principles of Program Analysis, Springer-Verlag.