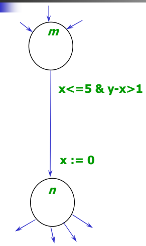
OUTLINE

- Introduction
  - Lecture 1: Motivation, examples, problems to solve
- Modeling and Verication of Timed Systems
  - Lecture 2: Timed automata, and timed automata in UPPAAL
  - Lecture 3: Symbolic verification: the core of UPPAAL
  - Lecture 4: Verification Options in UPPAAL
- Towards a Unified Framework
  - Lecture 5: Modeling, verification, real time scheduling, code synthesis
    From UPPAAL to TIMES

Verifying Timed Systems using Clock Constraints

Reachability Analysis and Constraint solving

Timed Automata: Semantics



State

(*location* , *clock-assignment*)

Transitions

(m , x=2.4 , y=3.1415) $\xrightarrow{1.1}$ (m, x=3.5 , y=4.241

(m , x=1.14 , y=3.1415) $\longrightarrow$ (n , x=0 , y=3.1415)

Other Verification Problems

- Timed Language Inclusion ☹
- Untimed Language Inclusion ☺
- (Un)Timed Bisimulation ☺
- Reachability Analysis ☺
- Optimal Reachability (synthesis problem) ☺
  - If a location is reachable, what is the minimal delay before reaching the location?

Reachability Problems

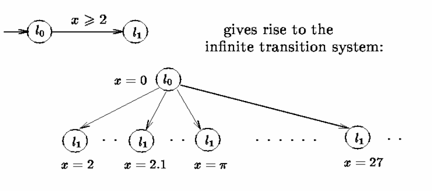
n is reachable from m if there is a sequence of transitions:

(m, x=r, y=s) $\xrightarrow{\quad}^{*}$ (n , x=r' , y=s')

Formalizing requirements

- Reachability properties: E<> Q
  - E<> P.stop
  - E<> (y>200)
- Invariant properties: A[] Q (not E<> not Q)
  - A[] not (P1.CS and P2.cs)
  - A[] (i < 100)
  - A[] (x>10 imply i>100)
    - After 10:00AM, i should be larger than 100
- Bounded Liveness Properties: F1 $\rightarrow_{<=10}$ F2
  - A[](f1 and x>10 imply f2)

Infinite State Space!



However, the reachability problem is decidable ☺ Alur&Dill 1991

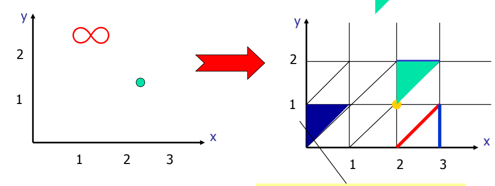
Alur and Dill 90

Finite Partitioning with "Regions"

Region: From infinite to finite

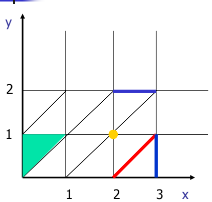
Concrete State
(n, x=2.2, y=1.5)

Symbolic state (region)
(n,)



An equivalence class (i.e. a *region*)
There are only *finite* many such!!

Region equivalence (Intuition)



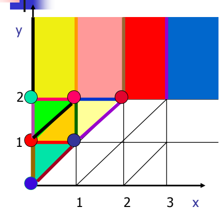
u ≅ v iff u and v satisfy exactly the same set of constraints in the form of
xi ~ m and xi-xj ~ n
where ~ is in {<,>,≤,≥}
and m,n < MAX

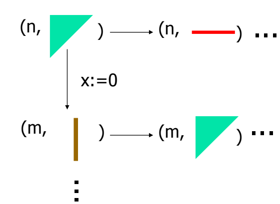
This is not quite correct;
we need to consider the MAX
more carefully

Region equivalence: Definition *[Alur and Dill 1990]*

- u,v are clock assignments
- u≈v iff
  - For all clocks x, either both u(x)>Cx and v(x)>Cx or ⌊u(x)⌋=⌊v(x) ⌋ (the same integer part)
  - For all clocks x, if u(x)<=Cx,
    {u(x)}=0 iff {v(x)}=0
  - For all clocks x, y, if u(x)<=Cx and u(y)<=Cy
    {u(x)}<= {u(y)} iff {v(x)}<= {v(y)}

Regions
Finite partitioning of state space



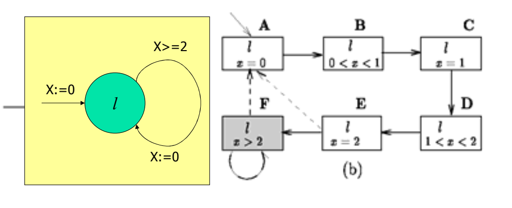


OBS: there are only
Finite many regions

An Important Theorem for Region Equivalence

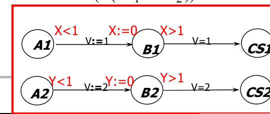
- u≈v implies
  - u(x:=0) ≈ v(x:=0)
  - u+n ≈ v+n for all natural number n
  - for all d<1: u+d ≈ v+d' for some d'<1
- that is, 'region equivalence' is preserved by "addition" and reset
- in fact, it is also preserved by "subtraction" if clock values are 'bounded'

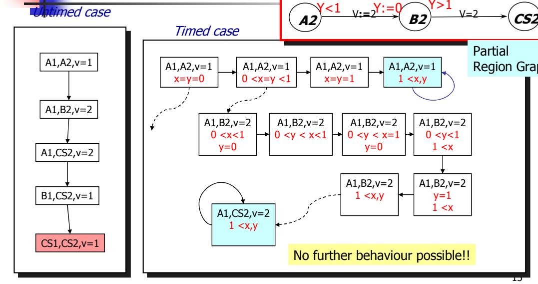
Region graph of a simple timed automata



Fischers again

$AG(\neg(CS_1 \wedge CS_2))$





Problems with Region Partitioning

- Too many 'regions'
- Sensitive to the maximal constants
  - e.g. x>1000000
- The number of regions is highly exponential in the number of clocks and the maximal constants (used to compare with clocks)

ZONES

The more efficient solution [UPPAAL, 1993 ~]

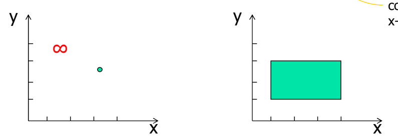
Symbolic Reachability
Using Clock Constraints

Zones: From infinite to finite

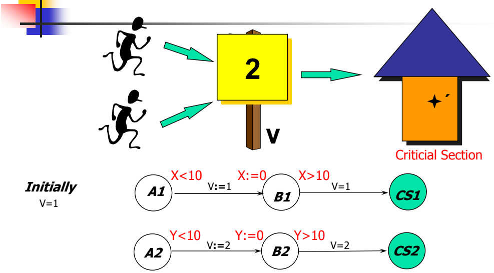
State
(n, x=3.2, y=2.5)

Symbolic state (zone)
(n, $1 \le x \le 4, 1 \le y \le 3$)

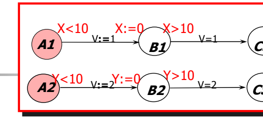
Zone:
conjunction of
x-y~n, x~n



Fischer's Protocol
analysis using zones



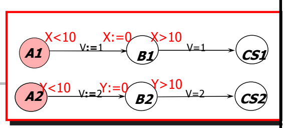
Fischers cont.



Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

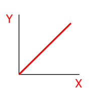
Fischers cont.



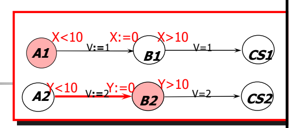
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



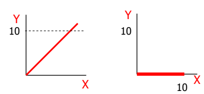
Fischers cont.



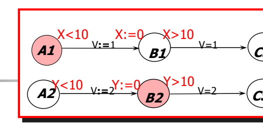
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



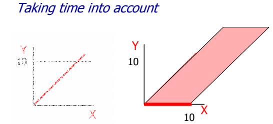
Fischers cont.



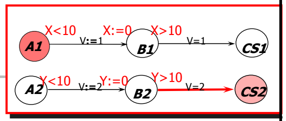
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



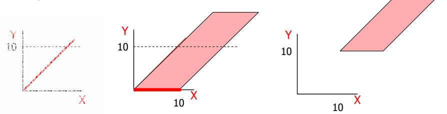
Fischers cont.



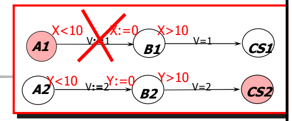
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



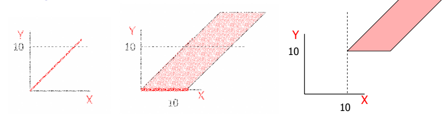
Fischers cont.



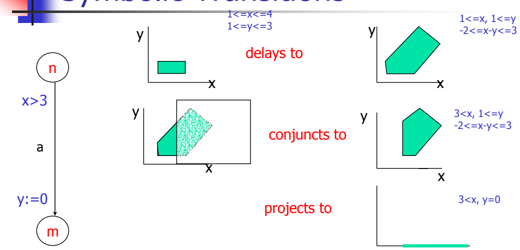
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



Symbolic Transitions



Thus (n,1<=x<=4,1<=y<=3) =a=> (m,3<x, y=0)

Zones = Conjuctive constraints

- A zone Z is a conjunctive formula:
  g_1 & g_2 & ... & g_n
  where g_i is a clock constraint:
  $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$
- Use a zero-clock x_0 (constant 0)
- A zone can be re-written as a set:
  $\{x_i - x_j \sim b_{ij} \mid \sim \text{ is } < \text{ or } \leq, i,j \leq n\}$
- This can be represented as a MATRIX, DBM (Difference Bound Matrices)

Solution set as semantics

- Let Z be a zone (a set of constraints)
- Let [Z]={u | u is a solution of Z}
  - The semantics

(We shall simply write Z instead [Z])

Operations on Zones

- Strongest post-condition (Delay): SP(Z) or Z↑
  - [Z↑] = {u+d| d ∈ R, u∈[Z]}
- Weakest pre-condition: WP(Z) or Z↓ (the dual of Z↑)
  - [Z↓] = {u| u+d∈[Z] for some d∈R}
- Reset: {x}Z or Z(x:=0)
  - [{x}Z] = {u[0/x] | u ∈[Z]}
- Conjunction
  - [Z&g]= [Z]∩[g]

An important theorem on Zones

- The set of zones is closed under all constraint operations (including x:=x-c or x:=x+c)
  - That is, the result of the operations on a zone is a zone
  - That is, there will be a zone (a finite object i.e a zone/constraints) to represent the sets: [Z↑], [Z↓], [{x}Z]

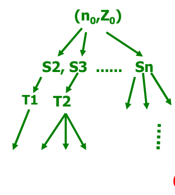
One-step reachability: Si→Sj

- Delay: (n,Z) → (n,Z') where Z'= Z↑ ∧ inv(n)
- Action: (n,Z) → (m,Z') where Z'= {x}(Z ∧g)

  if n $\xrightarrow{g \quad x:=0}$ m

- Successors(n,Z)={(m,Z') | (n,Z) →→(m,Z'), Z'≠Ø}
  - Sometime we write: (n,Z)→(m,Z') if (m,Z') is a successor of (n,Z)

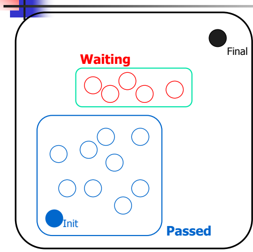
Now, we have a search problem



REACHABILITY ALGORITHM

Forward Rechability

Init -> Final ?



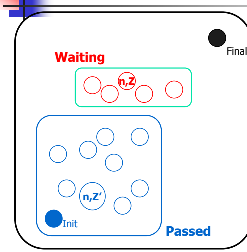
INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ **Z**
  (n,Z') in **Passed then STOP**
- **else** (explore) add
  successors(n,Z) to **Waiting**;
  Add (n,Z) to **Passed**

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Forward Rechability

Init -> Final ?



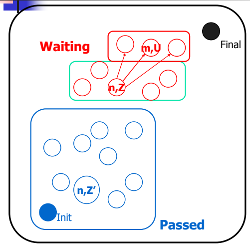
INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ Z
  (n,Z') in **Passed then STOP**

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Forward Rechability

Init -> Final ?



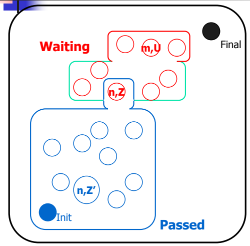
INITIAL Passed := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ Z
  (n,Z') in **Passed then STOP**
- **else** /explore/ add
  successors(n,Z) to **Waiting**;

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Forward Rechability

Init -> Final ?



INITIAL Passed := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ Z
  (n,Z') in **Passed then STOP**
- **else** /explore/ add
  successors(n,Z) to **Waiting**;
  Add (n,Z) to **Passed**

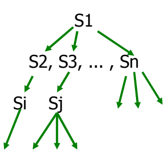
UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Two more operations on Zones

- Inclusion checking: $Z_1 \subseteq Z_2$
  - solution sets
- Emptiness checking: $Z = Ø$
  - no solution

All Operations on Zones
(needed for reachability analysis)

- Transformation
  - Conjunction
  - Post condition (delay)
  - Reset
- Consistency Checking
  - Inclusion
  - Emptiness



EFFICIENT IMPLEMENTATION

Canonical Datastructures for Zones

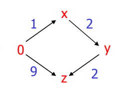
Difference Bounded Matrices Bellman 1958, Dill 1989

Inclusion

Z1

x<=1
y-x<=2
z-y<=2
z<=9

Graph

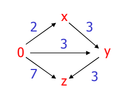


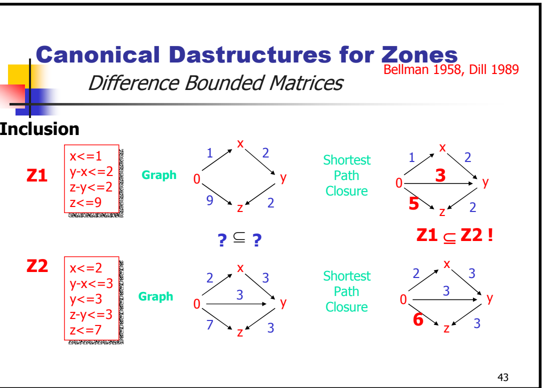
? ⊆ ?

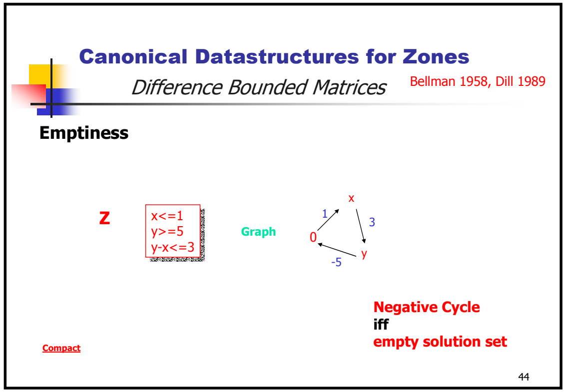
Z2

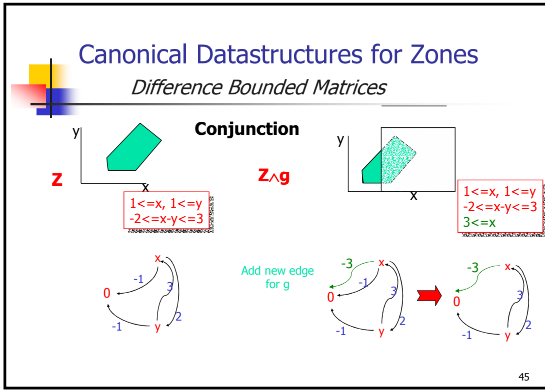
x<=2
y-x<=3
y<=3
z-y<=3
z<=7

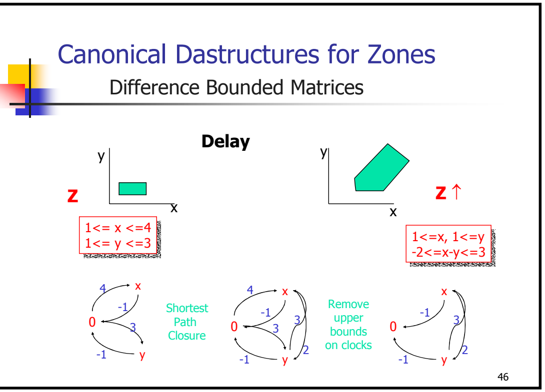
Graph

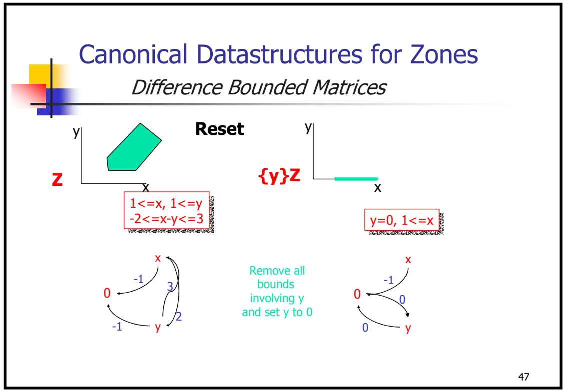


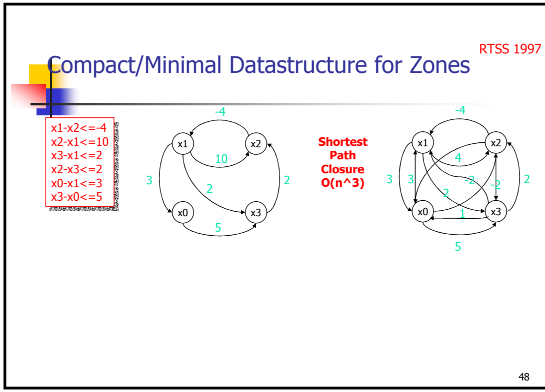

Canonical Dastructures for Zones
Difference Bounded Matrices
Bellman 1958, Dill 1989
Inclusion
Z1
x<=1
y-x<=2
z-y<=2
z<=9
Graph
Shortest Path Closure
? ⊆ ?
Z1 ⊆ Z2 !
Z2
x<=2
y-x<=3
y<=3
z-y<=3
z<=7
Graph
Shortest Path Closure
43


Canonical Datastructures for Zones
Difference Bounded Matrices
Bellman 1958, Dill 1989
Emptiness
Z
x<=1
y>=5
y-x<=3
Graph
Negative Cycle
iff
empty solution set
Compact
44


Canonical Datastructures for Zones
Difference Bounded Matrices
Conjunction
Z
1<=x, 1<=y
-2<=x-y<=3
Z∧g
1<=x, 1<=y
-2<=x-y<=3
3<=x
Add new edge for g
45


Canonical Dastructures for Zones
Difference Bounded Matrices
Delay
Z
1<= x <=4
1<= y <=3
Z ↑
1<=x, 1<=y
-2<=x-y<=3
Shortest Path Closure
Remove upper bounds on clocks
46

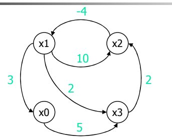

Canonical Datastructures for Zones
Difference Bounded Matrices
Reset
Z
1<=x, 1<=y
-2<=x-y<=3
{y}Z
y=0, 1<=x
Remove all bounds involving y and set y to 0
47


Compact/Minimal Datastructure for Zones
RTSS 1997
x1-x2<=-4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5
Shortest Path Closure O(n^3)
48

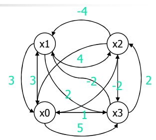
RTSS 1997

Compact/Minimal Data Structure for Zones

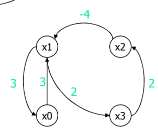
x1-x2<=4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5



Shortest Path Closure O(n^3)



Shortest Path Reduction O(n^3)



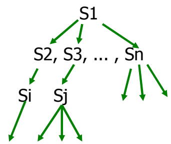
Canonical wrt =
Space worst O(n^2)
practice O(n)

COMPLEXITY

- Computing the shortest path closure, the cannonical form of a zone: $O(n^3)$ [Dijkstra's alg.]
- Run-time complexity, mostly in O(n)
  (when we keep all zones in cannonical form)

All Operations on Zones
(needed for reachability analysis)

- Transformation
  - Conjunction
  - Post condition (delay)
  - Reset
- Consistency Checking
  - Inclusion
  - Emptiness



How about termination?

We need the normalization operation according to the maximal constant